WNE Linear Algebra Final Exam
Series B

9 February 2017

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem.**

Problem 1.
Let $v_1 = (1, 1, 1, 2), v_2 = (1, 2, 3, 3), v_3 = (2, 0, t, 2)$ be vectors in $\mathbb{R}^4$.

a) for which $t \in \mathbb{R}$ vectors $v_1, v_2, v_3 \in \mathbb{R}^4$ are linearly independent?
b) find a system of linear equations which set of solutions is equal to $\mathrm{lin}(v_1, v_2, v_3)$ for $t = 0$.

Problem 2.
Let $W \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + 2x_2 + 3x_3 - x_4 + x_5 = 0 \\ x_1 + 2x_2 + 4x_3 + 3x_5 = 0 \\ 2x_1 + 5x_2 + 7x_3 - x_4 + 3x_5 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace W and the dimension of W,
b) complete the basis $\mathcal{A}$ to a basis $\mathcal{B}$ of $\mathbb{R}^5$ and find coordinates of $w = (1, 2, 0, 0, 0) \in \mathbb{R}^5$ relative to $\mathcal{B}$.

Problem 3.
Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the formula

$$\varphi((x_1, x_2, x_3)) = (tx_1, x_1 + 5x_2 - 6x_3, x_1 + 3x_2 - 4x_3).$$

a) for $t = 2$ find matrix $C \in M(3\times 3; \mathbb{R})$ such that matrix $C^{-1}M(\varphi)_{st}^{st}C$ is diagonal,
b) find all $t \in \mathbb{R}$ for which there exist a basis $\mathcal{A}$ of $\mathbb{R}^3$ such that $M(\varphi)_{\mathcal{A}}^{\mathcal{A}} = \begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q \end{bmatrix}$, where $p, q \in \mathbb{R}$.

Problem 4.
Let $\mathcal{A} = ((2, 0, 1), (0, 1, 0), (1, 0, 1))$ be an ordered basis of $\mathbb{R}^3$. The linear transformation $\psi\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is given by the matrix $M(\psi)_{\mathcal{A}}^{\mathcal{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

a) find $M(\psi)_{\mathcal{A}}^{st}$,
b) find formula of $\psi \circ \psi$.

Problem 5.
Let $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 2x_1 + x_2 - x_3 = 0\}$ be a subspace of $\mathbb{R}^3$.

a) find an orthonormal basis of $V^{\perp}$,
b) compute the orthogonal projection of $w = (0, 0, 6)$ onto V.

Problem 6.
Let

$$A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 1 \\ 2 & 7 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) compute matrix BA,
b) compute $\det(A^{-1}B^3 + B^4)$.

Problem 7.
Let $L \subset \mathbb{R}^3$ be an affine line given by the system of linear equations

$$\begin{cases} x_1 - x_3 = 1 \\ x_2 - 2x_3 = -1 \end{cases}$$

a) find a parametrization of L,
b) find an equation of the affine plane perpendicular to L passing through $(0, 1, 1)$.

Problem 8.
Consider the following linear programming problem $7x_2 - 5x_3 - 2x_5 \to \min$ in the standard form with constraints

$$\begin{cases} x_1 - x_2 + x_3 + x_4 = 2 \\ 3x_1 - 2x_2 + 2x_3 + x_5 = 5 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the sets $\mathcal{B}_1 = \{2, 3\}, \mathcal{B}_2 = \{2, 4\}, \mathcal{B}_3 = \{4, 5\}$ are basic? Which basic sets are feasible?
b) solve the linear programming problem using simplex method.